THE *BISECTION* METHOD

Introduction

- Bisection Method:

• Bisection Method is a numerical method in Mathematics to find a root of a given *function*

Introduction (cont.)

- *Root* of a function:

  - Root of a function $f(x) = 0$, is value a such that:

    - $f(a) = 0$

Introduction (cont.)

- Example:

Function: $f(x) = x^2 - 4$

Roots: $x = -2$, $x = 2$

Because:

$$f(-2) = (-2)^2 - 4 = 4 - 4 = 0$$

$$f(2) = (2)^2 - 4 = 4 - 4 = 0$$

A Mathematical Property

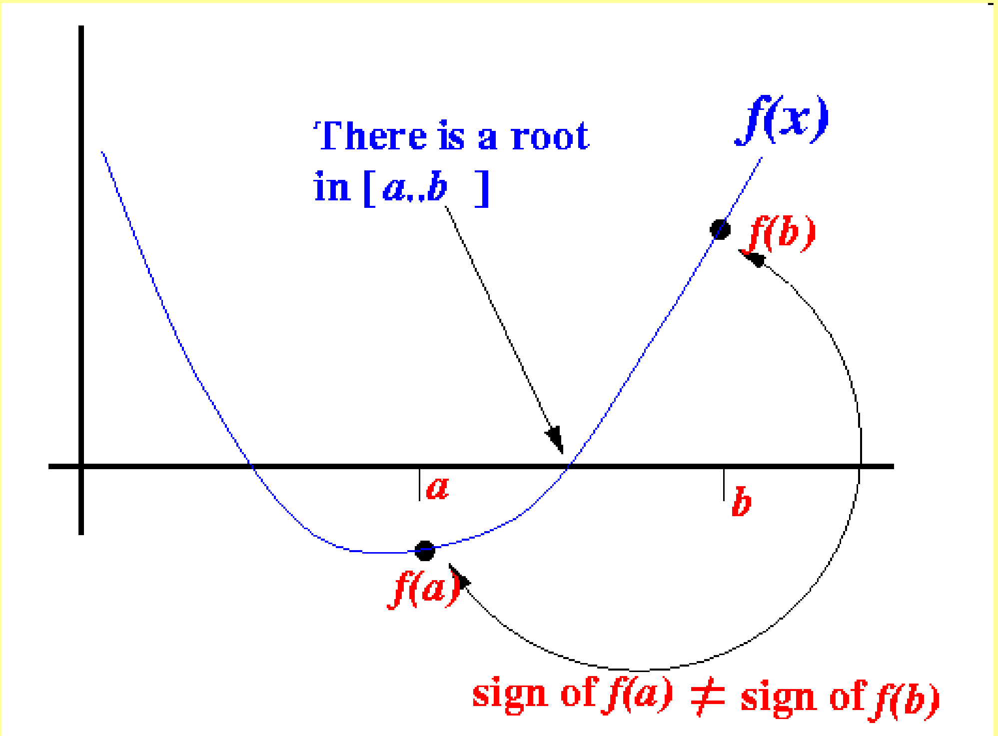
- Well-known Mathematical Property:

  - If a function $f(x)$ is continuous on the interval $[a,b]$ and sign of $f(a) \neq$ sign of $f(b)$, then

    - There is a value $c \in [a..b]$ such that: $f(c) = 0$
    I.e., there is a root c in the interval $[a..b]$

A Mathematical Property (cont.)

- Example:



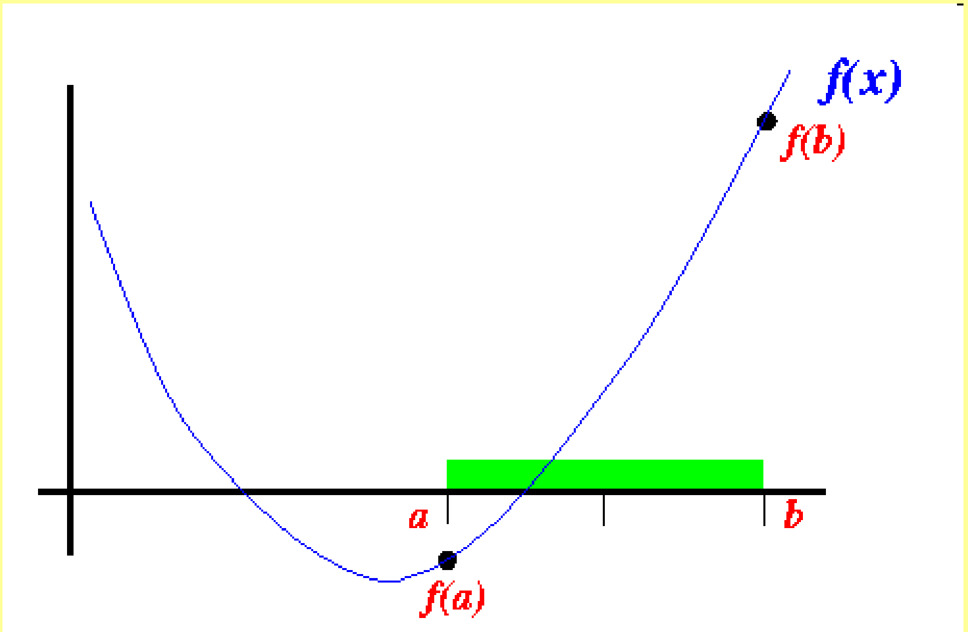
The *Bisection* Method

- The **Bisection Method** is a *successive* approximation method that narrows down an interval that contains a root of the function *f(x)*.
- The Bisection Method is *given* an initial interval [*a*..*b*] that contains a root (We can use the property sign of *f(a)* ≠ sign of *f(b)* to find such an initial interval).
- The Bisection Method will *cut the interval* into 2 halves and check which half interval contains a root of the function.
- The Bisection Method will keep *cut the interval* in halves until the resulting interval is extremely small

The root is then *approximately equal* to *any value* in the final (very small) interval.

The *Bisection* Method (cont.)

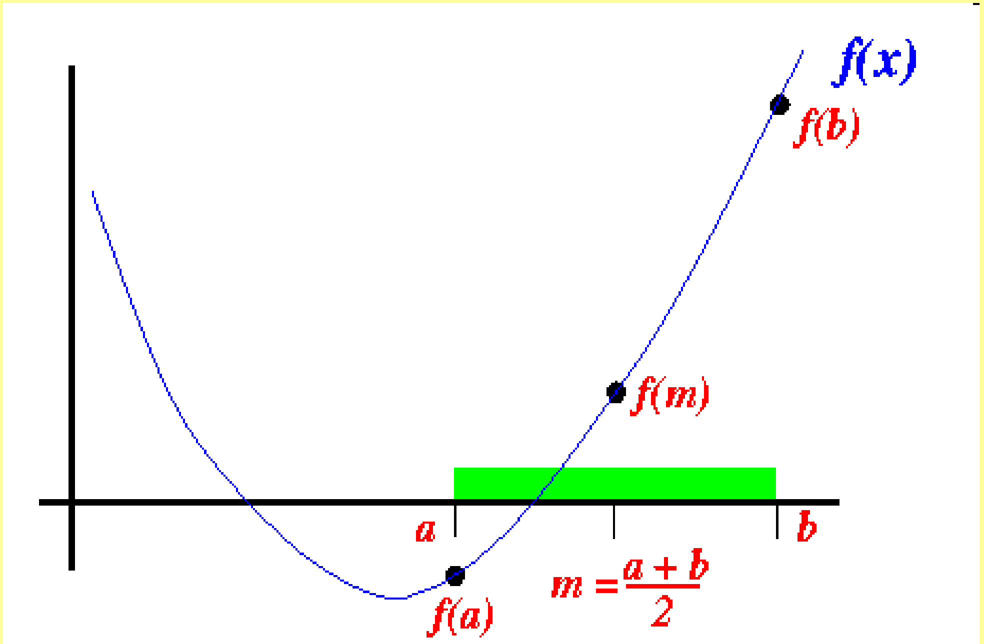
- Example:
  - Suppose the interval [*a*..*b*] is as follows:



The *Bisection* Method (cont.)

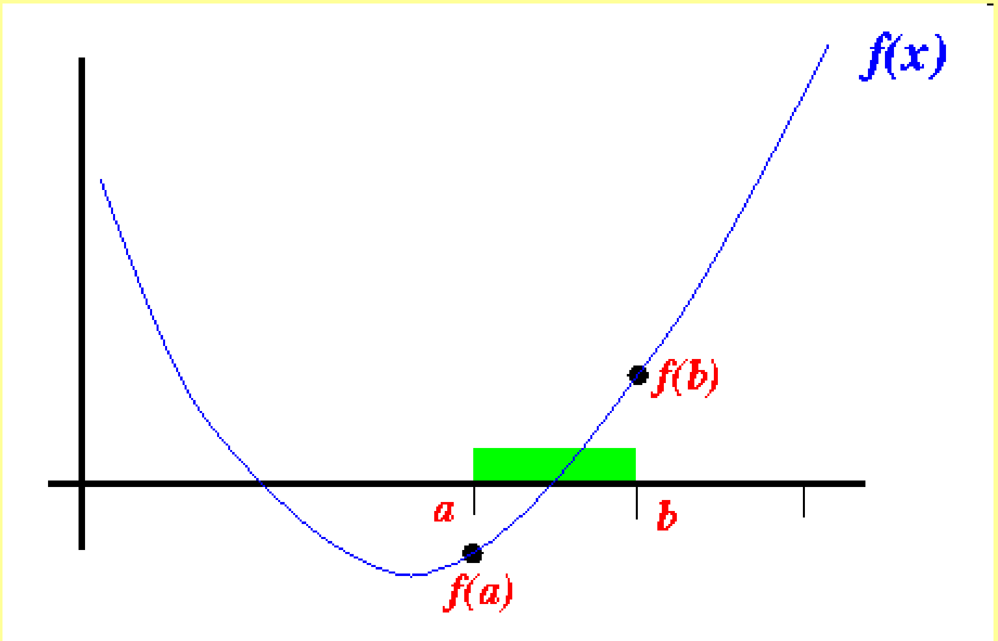
- We cut the interval [*a*..*b*] in the middle:

$m = (a+b)/2$



The *Bisection* Method (cont.)

- Because sign of *f(m)* ≠ sign of *f(a)* , we *proceed* with the search in the *new* interval [*a*,*b*]:



The *Bisection* Method (cont.)

We can use this statement to change to the new interval:

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b = m;
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The *Bisection* Method (cont.)

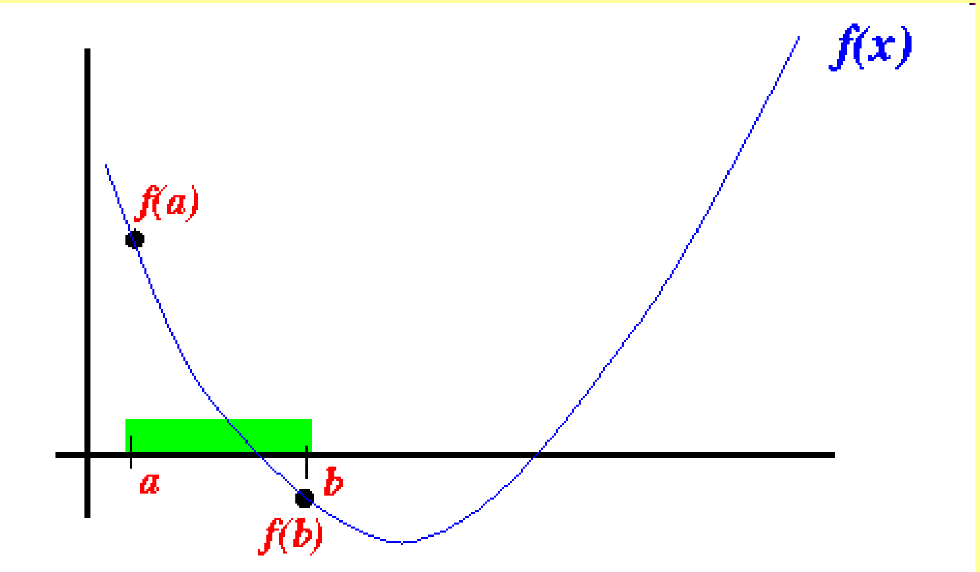
- In the above example, we have changed the end point *b* to obtain a smaller interval that still contains a root

  In other cases, we may need to changed the end point *b* to obtain a smaller interval that still contains a root

The *Bisection* Method (cont.)

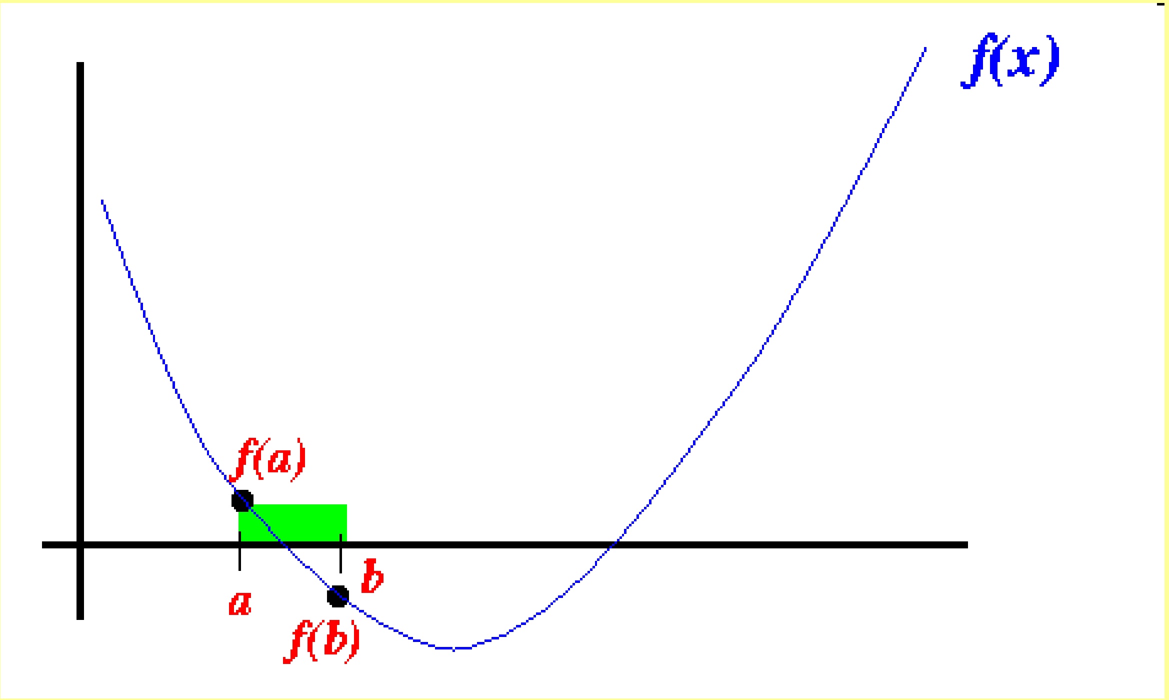
- Here is an example where you have to change the end point *a*:

  - Initial interval [*a*..*b*]:



The *Bisection* Method (cont.)

- After cutting the interval in half, the root is contained in the right-half, so we have to change the end point a:



Example 1 : solve the equation

$$x^3 - 1.1x^2 + 4x - 4.4 = 0$$

correct up to 2 significant figures using bisection method.

- Solution:

Here, $f(x) = x^3 - 1.1x^2 + 4x - 4.4$

$$f(1) = 1^3 - 1.1 * 1^2 + 4 * 1 - 4.4 = -0.5 < 0$$
$$f(2) = 2^3 - 1.1 * 2^2 + 4 * 2 - 4.4 = 7.2 > 0$$

Hence $f(1) * f(2) < 0$

Hence the required root is in $[1,2] = [a, b]$

i.e. $x \in [1,2]$, $for\ which\ \ f(x) = 0$

Taking $a = 1$ & $b = 2$ we have following.

N	a_n	b_n	$x_{n+1} = \frac{a_n + b_n}{2}$	$f(a_n)$	$f(b_n)$	$f(x_{n+1})$

solution:

N	a_n	b_n	$x_{n+1} = \frac{a_n + b_n}{2}$	$f(a_n)$	$f(b_n)$	$f(x_{n+1})$
0	1	2	1.5	-0.5	7.2	2.5
1	1	1.5	1.25	-0.5	2.5	0.834375
2	1	1.25	1.125	-0.5	0.834375	0.131641
3	1	1.125	1.0625	-0.5	0.131641	-0.19233
4	1.0625	1.125	1.09375	-0.19233	0.131641	-0.03248
5	1.09375	1.125	1.109375	-0.03248	0.131641	0.049038
6	1.09375	1.109375	1.101563	-0.03248	0.049038	0.008146
7	1.09375	1.101563	1.097656	-0.03248	0.008146	-0.0122
8	1.097656	1.101563	1.099609	-0.0122	0.008146	-0.00203
9	1.099609	1.101563	1.100586	-0.00203	0.008146	0.003053

- Example-2

Solve the equation $x^3 - 9x + 1 = 0$ for the roots lying between 2 and 3 correct up to 3 significant figures using bisection method.

- Example-3

Solve the equation $x + \ln x = 2$ for the roots lying between 1 and 2 correct up to 2 decimal places using bisection method.

Example 2 : solution

N	a_n	b_n	$x_{n+1} = \frac{a_n + b_n}{2}$	$f(a_n)$	$f(b_n)$	$f(x_{n+1})$
0	2	3	2.5	-9	1	-5.875
1	2.5	3	2.75	-5.875	1	-2.95313
2	2.75	3	2.875	-2.95313	1	-1.11133
3	2.875	3	2.9375	-1.11133	1	-0.09009
4	2.9375	3	2.96875	-0.09009	1	0.446259
5	2.9375	2.96875	2.953125	-0.09009	0.446259	0.175922
6	2.9375	2.953125	2.945313	-0.09009	0.175922	0.042378
7	2.9375	2.945313	2.941406	-0.09009	0.042378	-0.02399
8	2.941406	2.945313	2.943359	-0.02399	0.042378	0.00916
9	2.941406	2.943359	2.942383	-0.02399	0.00916	-0.00742

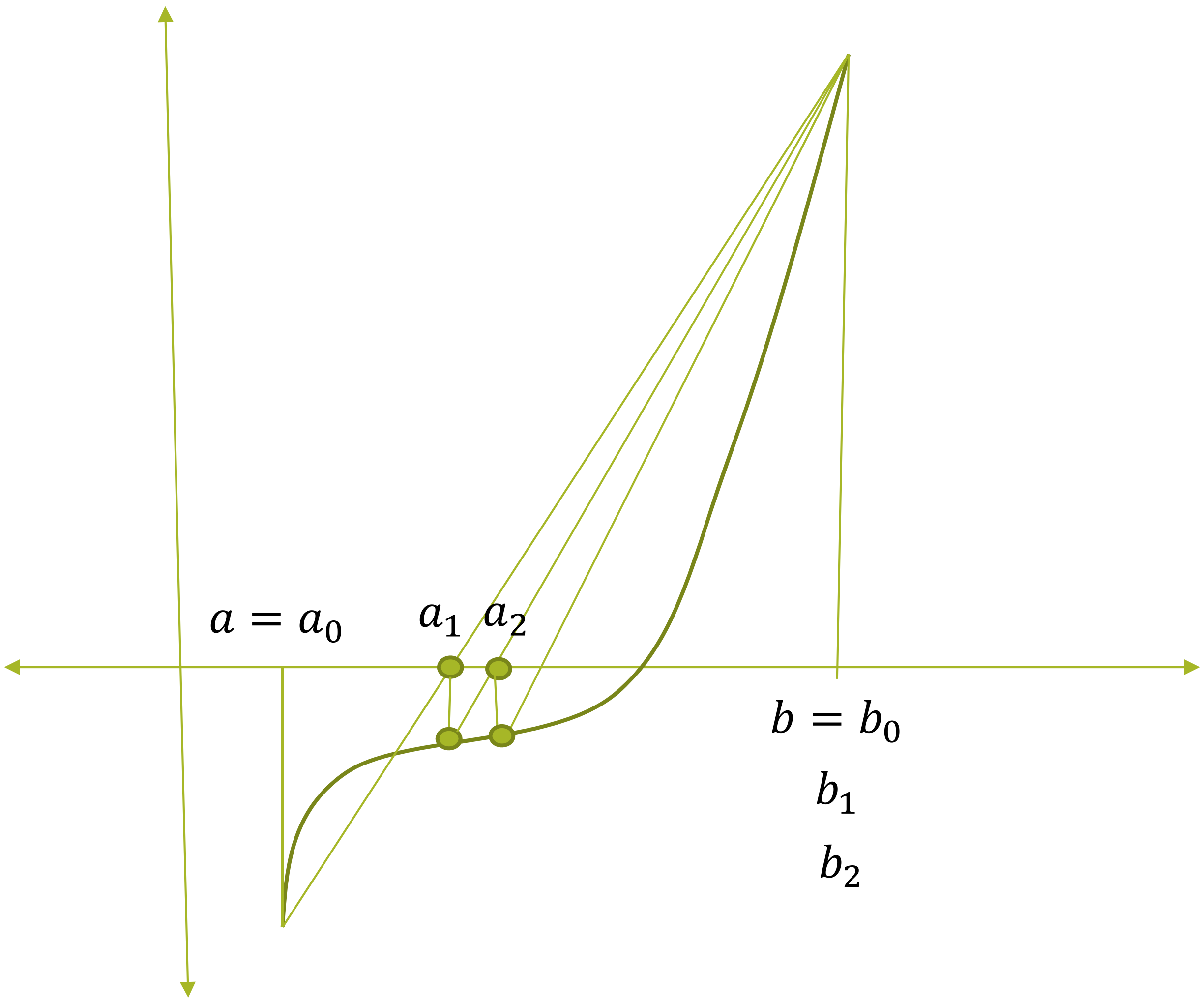
Solution :3

N	a_n	b_n	$x_{n+1} = \frac{a_n + b_n}{2}$	$f(a_n)$	$f(b_n)$	$f(x_{n+1})$
0	1	2	1.5	-1	0.693147	-0.09453
1	1.5	2	1.75	-0.09453	0.693147	0.309616
2	1.5	1.75	1.625	-0.09453	0.309616	0.110508
3	1.5	1.625	1.5625	-0.09453	0.110508	0.008787
4	1.5	1.5625	1.53125	-0.09453	0.008787	-0.04267
5	1.53125	1.5625	1.546875	-0.04267	0.008787	-0.01689
6	1.546875	1.5625	1.554688	-0.01689	0.008787	-0.00404
7	1.554688	1.5625	1.558594	-0.00404	0.008787	0.002378
8	1.554688	1.558594	1.556641	-0.00404	0.002378	-0.00083
9	1.556641	1.558594	1.557617	-0.00083	0.002378	0.000774

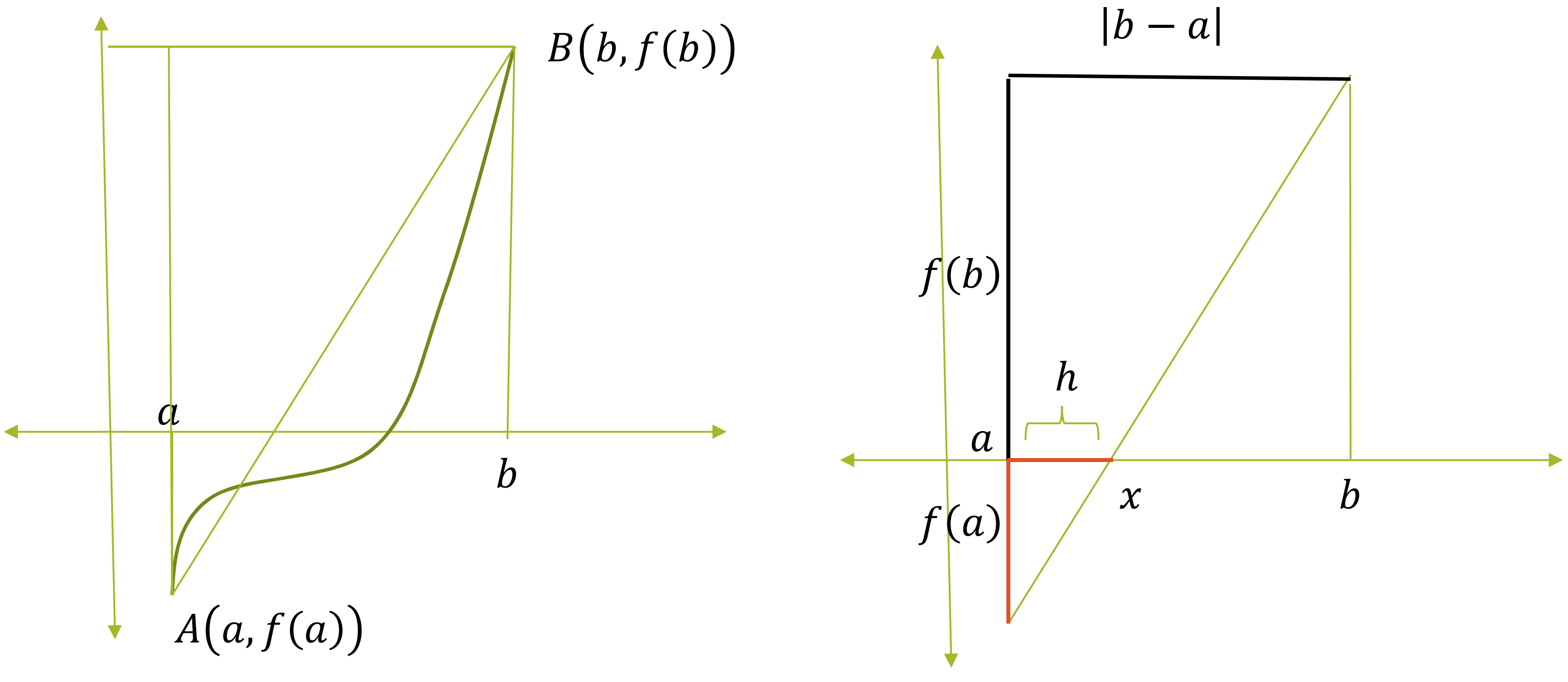
METHOD OF FALSE POSITION

(REGULA FALSI METHOD)

Process:



Process:



Process:

From the given figure we can see that triangle ΔOAB &ΔOCD are similar.

Hence,

$$\frac{AB}{CD} = \frac{OA}{OD}$$

$$\frac{h}{|b-a|} = \frac{|f(a)|}{|f(a)| + |f(b)|}$$

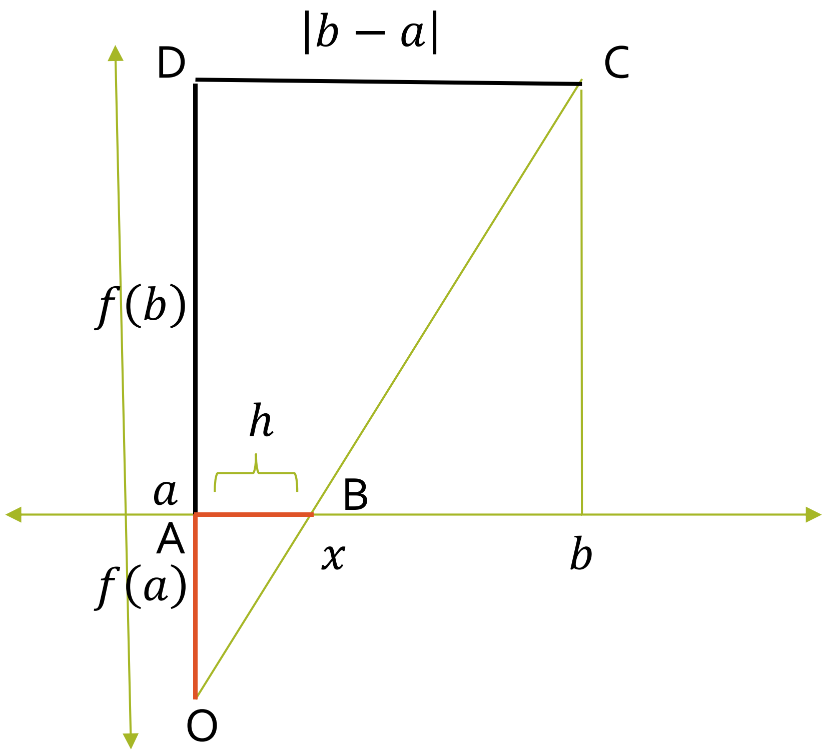
$$h = \frac{|f(a)| \; * |b-a|}{|f(a)| + |f(b)|}$$

$$x = a + h$$

For iterations

$$h_n = \frac{|f(a_n)| \; * |b_n - a_n|}{|f(a_n)| + |f(b_n)|}$$

$$x_{n+1} = a_n + h_n$$



Steps :

- Equation is given make appropriate Function from this call it $f(x)$
- Find a & b in which root is lying between.
- Follow the process …

n	a_n	b_n	$f(a_n)$	$f(b_n)$	$\|b_n - a_n\|$	h_n	x_{n+1}	$f(x_{n+1})$
0	a_0	b_0	$f(a_0)$	$f(b_0)$	$\|b_0 - a_0\|$	h_0	x_{0+1}	$f(x_{0+1})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Example 1: using Regula-falsi (RF) method find the root of function $x^3 + 2x - 2 = 0$ correct up to 3 significant figures.

- Solution:

Here, $f(x) = x^3 + 2x - 2$

$$f(0) = -2 < 0$$
$$f(1) = 1 > 0$$

Hence $f(0) * f(1) < 0$

Hence the required root is in $[0,1] = [a, b]$

i.e. $x \in [0,1]$, $for\ which\ \ f(x) = 0$

Taking $a = 0$ & $b = 1$ we have following.

Solution 1:

n	a_n	b_n	$f(a_n)$	$f(b_n)$	$\lvert b_n - a_n \rvert$	h_n	x_{n+1}	$f(x_{n+1})$
0	0.00000	1.00000	-2.00000	1.00000	1.00000	0.66667	0.66667	-0.37037
1	0.66667	1.00000	-0.37037	1.00000	0.33333	0.09009	0.75676	-0.05311
2	0.75676	1.00000	-0.05311	1.00000	0.24324	0.01227	0.76902	-0.00716
3	0.76902	1.00000	-0.00716	1.00000	0.23098	0.00164	0.77066	-0.00096
4	0.77066	1.00000	-0.00096	1.00000	0.22934	0.00022	0.77088	-0.00013
5	0.77088	1.00000	-0.00013	1.00000	0.22912	0.00003	0.77091	-0.00002
6	0.77091	1.00000	-0.00002	1.00000	0.22909	0.00000	0.77092	0.00000
7	0.77092	1.00000	0.00000	1.00000	0.22908	0.00000	0.77092	0.00000

- Example-2

Solve the equation $xlnx - 1 = 0$ by RF method, correct upto 3 decimal places.

- Example-3

Solve the equation $\sin x + \cos x = 1$ by RF method correct up to 4 significant places.

Solution 2:

n	a_n	b_n	$f(a_n)$	$f(b_n)$	$\|b_n - a_n\|$	h_n	x_{n+1}	$f(x_{n+1})$
0	1.00000	2.00000	-1.00000	0.38629	1.00000	0.72135	1.72135	-0.06512
1	1.72135	2.00000	-0.06512	0.38629	0.27865	0.04020	1.76155	-0.00263
2	1.76155	2.00000	-0.00263	0.38629	0.23845	0.00161	1.76316	-0.00010
3	1.76316	2.00000	-0.00010	0.38629	0.23684	0.00006	1.76322	0.00000
4	1.76322	2.00000	0.00000	0.38629	0.23678	0.00000	1.76322	0.00000
5	1.76322	2.00000	0.00000	0.38629	0.23678	0.00000	1.76322	0.00000
6	1.76322	2.00000	0.00000	0.38629	0.23678	0.00000	1.76322	0.00000
7	1.76322	2.00000	0.00000	0.38629	0.23678	0.00000	1.76322	0.00000

Solution 1:

n	a_n	b_n	$f(a_n)$	$f(b_n)$	$\|b_n - a_n\|$	h_n	x_{n+1}	$f(x_{n+1})$
0	1.50000	2.00000	0.06823	-0.50685	0.50000	0.05932	1.55932	0.01141
1	1.55932	2.00000	0.01141	-0.50685	0.44068	0.00970	1.56902	0.00177
2	1.56902	2.00000	0.00177	-0.50685	0.43098	0.00150	1.57052	0.00027
3	1.57052	2.00000	0.00027	-0.50685	0.42948	0.00023	1.57075	0.00004
4	1.57075	2.00000	0.00004	-0.50685	0.42925	0.00004	1.57079	0.00001
5	1.57079	2.00000	0.00001	-0.50685	0.42921	0.00001	1.57080	0.00000
6	1.57080	2.00000	0.00000	-0.50685	0.42920	0.00000	1.57080	0.00000
7	1.57080	2.00000	0.00000	-0.50685	0.42920	0.00000	1.57080	0.00000

THANK YOU